# Covariant axial form factor of the nucleon in a chiral constituent quark model

L.Ya. Glozman [a,b], M. Radici [b], R.F. Wagenbrunn [a], S. Boffi [b], W. Klink [c], and W. Plessas [a]

[a] *Institut für Theoretische Physik, Universität Graz, Universitätsplatz 5, A-8010 Graz, Austria*

[b] *Dipartimento di Fisica Nucleare e Teorica, Università di Pavia, and Istituto Nazionale di Fisica Nucleare, Sezione di Pavia, I-27100 Pavia, Italy*

[c] *Department of Physics and Astronomy, University of Iowa, Iowa City, IA 52242, USA*

The axial form factor $G_A$ of the nucleon is investigated for the Goldstone-boson-exchange constituent quark model using the point-form approach to relativistic quantum mechanics. The results, being covariant, show large contributions from relativistic boost effects. The predictions are obtained directly from the quark-model wave functions, without any further input such as vertex or constituent-quark form factors, and fall remarkably close to the available experimental data.

*PACS*: 12.39-x; 13.10.+q; 14.20.Dh
*Keywords*: axial form factor, constituent quark model, point-form relativistic quantum mechanics

## 1 Introduction

Low-energy hadron phenomena in the nonperturbative regime of quantum chromodynamics (QCD) are suitably described in terms of effective degrees of freedom within models incorporating the relevant properties of QCD. In particular, the phenomenon of spontaneous breaking of chiral symmetry (SB$\chi$S) is known to reduce the original $\mathrm{SU}(3)_\mathrm{L} \times \mathrm{SU}(3)_\mathrm{R}$ symmetry of QCD to an $\mathrm{SU}(3)_\mathrm{V}$ vector symmetry. As a first consequence, the practically massless current quarks acquire a dynamical mass related to the nonzero value of the quark condensate [1]. Such a dynamical mass can be viewed as the mass of quasiparticles, which can be interpreted as the constituent quarks commonly used in quark models [2]. Just recently, this dynamical-mass generation has been established by lattice QCD calculations, with the result that the constituent

*Preprint submitted to Elsevier Preprint* *8 March 2002*

mass approaches values of 300 - 400 MeV at small (Euclidean) momenta [3]. Another important consequence of SB$\chi$S is that collective quark-antiquark excitations can be identified with Goldstone-boson fields [1] and these Goldstone bosons should be coupled to constituent quarks [4]. The latter should thus consistently be included as proper effective degrees of freedom in the formulation of constituent quark models (CQM). Consequently, Goldstone-boson exchange (GBE) becomes responsible for mediating the (residual) interaction between constituent quarks.

A chiral quark model built up in this spirit for light-flavor baryons was suggested in Ref. [5]. It was further elaborated and then parametrized in a semirelativistic framework [6]. This version of the GBE chiral quark model is used in the present work. It relies on a three-quark Hamiltonian containing a relativistic kinetic-energy operator and a linear confinement, whose strength is taken according to the string tension of QCD. The hyperfine interaction of the constituent quarks is derived from GBE. It is realized by the exchange of octet plus singlet pseudoscalar mesons, where only the spin-spin components are taken into account. The inclusion of the other force components as well as the consideration of possible vector and scalar exchanges are under investigation [7]. However, the spin-spin part is the most important ingredient for the hyperfine interaction and indeed it already provides for a very reasonable description of the low-energy spectra of all light and strange baryons. In particular, the specific spin-flavor dependence of the short-range part of the GBE interaction produces the correct level orderings of the lowest positive- and negative-parity excitations and thereby offers a convincing solution to a long-standing problem in baryon spectroscopy.

Beyond spectroscopy, however, a constituent quark model should in addition also provide for the description of other low-energy hadron phenomena, such as electromagnetic form factors and transitions, mesonic decays etc. The GBE constituent quark model, in the version of Ref. [6], has recently been applied, e.g., in a first study of the nucleon electromagnetic form factors (including proton and neutron charge radii and magnetic moments) [8]. ¿From this investigation it has in particular turned out that a proper treatment of relativistic effects in the three-quark system is most essential. In the present work we report on a study of the nucleon axial form factor. Again it is an immediate demand to follow an approach that allows for a strict observation of relativistic covariance. In order to reach this aim we have chosen to investigate the problem in the framework of relativistic Hamiltonian dynamics (i.e. Poincaré-invariant quantum mechanics).

Among the various forms of relativistic Hamiltonian dynamics that can be considered in terms of unitary representations of the Poincaré group, first discussed by Dirac [9], the point form [10,11] in particular offers some specific advantages. Specifically, the interactions are contained only in the 4-momentum operator $P^\mu$ that generates the space-time evolution through the covariant equation

$$P^\mu|\Psi\rangle = p^\mu|\Psi\rangle\,, \tag{1}$$

where $|\Psi\rangle$ is an element of the Hilbert space (for a system with a fixed number of particles). Therefore, the unitary representations $U(\Lambda)$ of Lorentz transformations $\Lambda$, consisting of boosts and spatial rotations of the wave functions $|\Psi\rangle$, contain no interactions at all and remain purely kinematic. The theory is thus manifestly covariant. Furthermore, the different $P^\mu$'s commute with each other so that they can be diagonalized simultaneously. Considering a three-body problem with constituent (quark) masses $m_i$ and individual 3-momenta $\vec{k}_i$, the interactions can be introduced through the so-called Bakamjian-Thomas (BT) construction [12], by adding to the free mass operator $M_0 = \sqrt{P_0^\mu P_{0,\mu}} = \sum_i \sqrt{\vec{k}_i^2 + m_i^2}$ an interaction part $M_I$ so that $M = \sqrt{P^\mu P_\mu} = M_0 + M_I$. Then also the 4-momentum operator gets split into a free part $P_0^\mu$ and an interaction part $P_I^\mu$:

$$P^\mu = P_0^\mu + P_I^\mu = MV^\mu = (M_0 + M_I)V^\mu. \tag{2}$$

Here $V^\mu$ is the 4-velocity of the system, which is *not* modified by the interactions (i.e., $V^\mu = V_0^\mu$). The mass operator $M$ with interactions must satisfy the following conditions

$$[V^\mu, M] = 0, \qquad U(\Lambda)MU^{-1}(\Lambda) = M \tag{3}$$

in order to fulfil the Poincaré algebra of the 4-momentum operators. In the center-of-momentum frame of the three-body system, for which $\vec{P} = \sum_i \vec{k}_i = 0$ and $V^\mu = (1,0,0,0)$, the stationary part of Eq. (1) can be identified with the eigenvalue problem solved in Ref. [6] for the GBE Hamiltonian $H = \sum_i \sqrt{\vec{k}_i^2 + m_i^2} + H_I = H_0 + H_I$. Therefore, the latter, even if including in $H_I$ a phenomenological confinement and an instantaneous hyperfine interaction, can be interpreted as a mass operator fulfilling all the necessary commutation relations of the Poincaré group. The corresponding eigenfunctions describe all possible states of the three-body system in the center-of-momentum frame.

Under an arbitrary Lorentz transformation, each quark spin gets rotated by a different Wigner rotation $R_{W_i}$, thus preventing the definition of a total spin for the rotated state. It is useful to introduce the so-called velocity states [13] by applying a particular Lorentz boost $B(v)$ to the eigenfunctions in the center-of-momentum frame. This boost takes the whole system from the rest frame to a four-velocity $v$ with new four-momenta $p_i = B(v)k_i$ for the individual quarks. Now, under any Lorentz transformation $\Lambda$, each individual quark spin and orbital angular momentum in the velocity states is rotated by the same Wigner rotation $R_W = B^{-1}(\Lambda v)\Lambda B(v)$ so that it is always possible to define a total spin in the same way as in nonrelativistic quantum mechanics. In practice, the point-form approach together with the use of velocity states allows for an exact calculation of all necessary transformations of the momentum dependences and of relativistic quark-spin rotations associated with proper Lorentz boosts of the three-quark wave functions.

In the case of electromagnetic reactions, the point-form electromagnetic current operator $J^\mu$ can be written in terms of irreducible tensor operators under the strongly interacting Poincaré group [11]. Thus, e.g., the nucleon charge and magnetic form factors can be obtained as reduced matrix elements of such an irreducible tensor operator in the Breit frame with the virtual photon momentum along the $\hat{z}$ axis, i.e. $q_B^\mu = (0, 0, 0, q)$. By using the eigenfunctions of the GBE Hamiltonian [6] boosted to velocity states in the Breit frame, these form factors have been calculated in Ref. [8], assuming a single-particle current operator for point-like quarks. This approach corresponds to a relativistic impulse approximation but specifically in the point form. It is conventionally called Point-Form Spectator Approximation (PFSA). Without introducing any further adjustable parameters (such as vertex cut-offs or quark form factors) the corresponding results have been found to fall remarkably close to existing experimental data for all elastic observables, i.e. proton and neutron electric as well as magnetic form factors and charge radii as well as magnetic moments.

Here, we report predictions of the nucleon axial form factor that, in contrast to the electromagnetic case, connects the proton wave function to the neutron one and, therefore, it represents a further test of the model wave functions. In the following Section we outline the calculation of axial current matrix elements in the point-form approach. In Section 3 we present the results and in Section 4 we discuss some of the main reasons why this approach appears so promising. We give a short summary in Section 5.

## 2 The axial form factor in the point-form approach

The axial-current matrix element between the initial and final nucleon states with 4-momenta $p$ and $p'$, and spins $s$ and $s'$, respectively, is defined as

$$\langle p', s'|A_a^\mu|p, s\rangle = \overline{u}(p', s')\left[G_A(Q^2)\gamma^\mu + \frac{1}{2M}G_P(Q^2)(p'^\mu - p^\mu)\right]\gamma_5 \tfrac{1}{2}\tau_a u(p, s), \quad (4)$$

where $M$ is the nucleon mass, $q^\mu = p'^\mu - p^\mu$, $Q^2 = \vec{q}^{\,2} - \omega^2 \geq 0$, $\tau_a$ is the isospin matrix with Cartesian index, and $u(p, s)$ is the usual Dirac spinor. Here, $G_A(Q^2)$ is the axial form factor and $G_P(Q^2)$ the induced pseudoscalar form factor. In the Breit frame, with the momentum transferred only along the $\hat{z}$ axis, we have

$$p_B^\mu = (E_B, 0, 0, -\tfrac{1}{2}|\vec{q}|), \qquad p_B'^\mu = (E_B, 0, 0, \tfrac{1}{2}|\vec{q}|)\,, \qquad E_B = \sqrt{M^2 + \tfrac{1}{4}\vec{q}^{\,2}}\,. \quad (5)$$

Therefore, one obtains

$$\begin{aligned}
\langle p_B', s'|A_a^0|p_B, s\rangle &= 0, \\
\langle p_B', s'|\vec{A}_a|p_B, s\rangle &= \chi_{s'}^\dagger \left[\frac{E_B}{M} G_A(Q^2)\vec{\sigma}_T \right. \\
&\qquad \left. + \left( G_A(Q^2) - \frac{\vec{q}^{\,2}}{4M^2} G_P(Q^2) \right) \vec{\sigma}_L \right] \tfrac{1}{2}\tau_a \chi_s\,, \qquad (6)
\end{aligned}$$

where

$$\vec{\sigma}_T = \vec{\sigma} - \hat{q}(\vec{\sigma}\cdot\hat{q})\,, \qquad \vec{\sigma}_L = \hat{q}(\vec{\sigma}\cdot\hat{q}) \qquad (7)$$

and $\chi_s$ is the two-component spinor of the nucleon.

The axial form factor $G_A(Q^2)$ is the only contribution to the transverse part of the current that is not affected by current conservation. Therefore, one can obtain $G_A$ by applying to $A_a^\mu$ the same PFSA approach as followed in Refs. [8,11] for the electromagnetic current $J^\mu$. In the following, we shall use for the initial nucleon state the neutron wave function and for the final nucleon state the proton wave function. Consequently, the isospin index $a$ in Eq. (6) may be suppressed whenever not needed. In the Breit frame, the matrix elements of the transverse components of the axial current $A^{i=1,2}$ can be connected to the reduced matrix elements of the corresponding irreducible tensor of the Poincaré group, i.e.

$$\langle p_B', s'|A^i|p_B, s\rangle = G^i_{s's}\,, \qquad i = 1, 2\,, \qquad (8)$$

with the following identifications from Eq. (6):

$$\begin{aligned}
G^1_{s's} &= \frac{E_B}{M} G_A \chi_{s'}^\dagger \sigma_x \chi_s = \frac{E_B}{M} G_A (\delta_{s',s+1} + \delta_{s',s-1})\,, \\
G^2_{s's} &= \frac{E_B}{M} G_A \chi_{s'}^\dagger \sigma_y \chi_s = -i\frac{E_B}{M} G_A (\delta_{s',s+1} - \delta_{s',s-1})\,. \qquad (9)
\end{aligned}$$

The combined invariance under parity and time reversal gives

$$G^1 = \begin{pmatrix} 0 & G \\ G & 0 \end{pmatrix}, \quad G^2 = -i \begin{pmatrix} 0 & G \\ -G & 0 \end{pmatrix}, \tag{10}$$

with $G$ real. The comparison with Eq. (9) thus gives

$$\frac{E_B}{M} G_A = G \, . \tag{11}$$

The calculation of the reduced matrix elements $G^i_{s's}$ in Eq. (8) is made in PFSA and follows the same lines of the formalism developed in Ref. [11] and already used in Ref. [8]. Then one has

$$\begin{aligned} G^i_{s's}(Q^2) = 3 \int & d\vec{k}_1 d\vec{k}_2 d\vec{k}_3 d\vec{k}'_1 d\vec{k}'_2 d\vec{k}'_3 \, \delta(\vec{k}_1 + \vec{k}_2 + \vec{k}_3) \, \delta(\vec{k}'_1 + \vec{k}'_2 + \vec{k}'_3) \\ & \times \psi^*_{s'}(\vec{k}'_1, \vec{k}'_2, \vec{k}'_3; \mu'_1, \mu'_2, \mu'_3) \; \psi_s(\vec{k}_1, \vec{k}_2, \vec{k}_3; \mu_1, \mu_2, \mu_3) \\ & \times D^{1/2}_{\lambda'_1 \mu'_1}{}^* [R_W(k'_1, B(v_{\rm out}))] \langle p'_1, \lambda'_1 | A^i_{[1]} | p_1, \lambda_1 \rangle D^{1/2}_{\lambda_1 \mu_1} [R_W(k_1, B(v_{\rm in}))] \\ & \times D^{1/2}_{\mu'_2 \mu_2} [R_W(k_2, B^{-1}(v_{\rm out}) B(v_{\rm in}))] D^{1/2}_{\mu'_3 \mu_3} [R_W(k_3, B^{-1}(v_{\rm out}) B(v_{\rm in}))] \\ & \times \delta^3 [k'_2 - B^{-1}(v_{\rm out}) B(v_{\rm in}) k_2] \delta^3 [k'_3 - B^{-1}(v_{\rm out}) B(v_{\rm in}) k_3] \, , \end{aligned} \tag{12}$$

where a summation is understood for repeated indices and the initial and final 4-velocities are $M v_{\rm in} = p_B$ and $M v_{\rm out} = p'_B$, respectively. In Eq. (12) $\psi_s$ is the nucleon wave function in the centre-of-momentum frame with $\vec{k}_i$ and $\mu_i$ being the individual quark momenta and spin projections, respectively, and $D^{1/2}$ is the standard rotation matrix. The single-quark axial-current matrix element has the form

$$\langle p'_i, \lambda'_i | A^\mu_{a\,[i]} | p_i, \lambda_i \rangle = g^q_A \bar{u}(p'_i, \lambda'_i) \gamma^\mu \gamma_5 \tfrac{1}{2} \tau_a u(p_i, \lambda_i), \tag{13}$$

where $u(p_i, \lambda_i)$ is the Dirac spinor of quark $i$ with momentum $p_i$ and spin projection $\lambda_i$, and $\tilde{q} = p'_i - p_i$ is the momentum transferred to a single quark. The quark axial charge is assumed to be $g^q_A = 1$, as for free bare fermions.

## 3 Results

The prediction for the axial form factor, calculated in relativistic PFSA as described in the previous Section, is given by the solid curve in Fig. 1. For

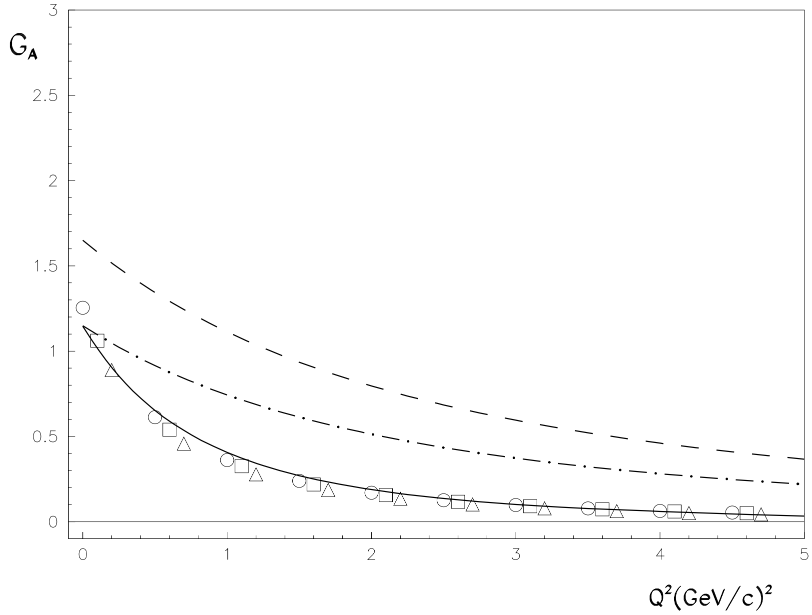


Fig. 1. Nucleon axial form factor $G_A$. Solid line: Fully relativistic PFSA result. Dashed line: Nonrelativistic result. Dot-dashed line: Result with relativistic current operator but without boosts. The experimental data are plotted following the dipole form of Eq. (14). Squares and circles correspond to different values for the axial mass $M_A$ from charged-pion electroproduction experiments; triangles correspond to the world average $M_A$ value from neutrino (antineutrino) scattering on protons and nuclei (see the text).

comparison, the dashed line represents the purely nonrelativistic result obtained if the nonrelativistic expression for the quark axial current is adopted and no boosts are applied to either the initial or final nucleon states. In this case, the axial constant is $g_A \equiv G_A(0) = 1.65$ and the marginal deviation from the value $\frac{5}{3}$ predicted with SU(6) harmonic oscillator wave functions is due to the small admixture of mixed-symmetry components in the nucleon wave functions of the GBE quark model of Ref. [6]. The dot-dashed line in Fig. 1 shows the result if the relativistic quark axial current is adopted but no boosts are applied (cf. Ref. [14]). Comparing this result to the full line, it becomes evident that at $Q^2 = 0$ the boosts do not contribute and the axial constant adopts the same value as in the complete relativistic calculation.

The experimental data are presented in Fig. 1 assuming the dipole form

$$G_A(Q^2) = \frac{g_A}{\left(1 + \dfrac{Q^2}{M_A^2}\right)^2}, \tag{14}$$

where the axial constant was taken to be $g_A = 1.255 \pm 0.006$, as obtained from $\beta$-decay experiments [15]. For the axial mass $M_A$ we used three different values, namely the world average $M_A = 1.069 \pm 0.016$ GeV from charged-pion electroproduction (represented by squares), the most recent value $M_A = 1.077 \pm 0.039$ GeV from the p(e,e$'\pi^+$)n experiment at Mainz [16] (represented by circles), and the world average $M_A = 1.032 \pm 0.036$ GeV from neutrino (antineutrino) scattering experiments on protons and nuclei [17] (represented by triangles).

At $Q^2 = 0$ the predicted value of the axial constant $g_A \equiv G_A(0) = 1.15$ is a bit lower than the experimental one. It is not yet clear which effect is responsible for this behaviour. We can think of a number of reasons (e.g., a quark axial constant $g_A^q$ different from 1, etc.), which, however, would require a series of further investigations going beyond the scope of this paper.

Similarly, the axial radius deduced from the slope of $G_A$ at $Q^2 = 0$ is also lower than the experimental one. For the GBE quark model of Ref. [6] one gets $< r_A^2 >^{1/2} = 0.520$ fm, whereas the experimental value is $< r_A^2 >^{1/2} = (0.635 \pm 0.023)$ fm, as extracted from pion electroproduction [16], or $< r_A^2 >^{1/2} = (0.65 \pm 0.07)$ fm, as deduced from neutrino experiments [17].

All the results were calculated with the nucleon wave functions from the GBE quark model as the only input. Point-like constituent quarks were assumed and no further phenomenological parameters were introduced. For the $Q^2$-range shown in Fig. 1, the PFSA results just fall on top of the experimental data. These results are similarly remarkable as before in the case of elastic electromagnetic form factors, where the PFSA results also came quite close to the experimental data [8]. The comparison with the nonrelativistic result (dashed line) shows a large discrepancy. On the one hand this is due to the use of a nonrelativistic current operator and on the other hand it misses the relativistic boost effects. That the latter are of considerable importance can be deduced from the comparison with the 'intermediate' result represented by the dot-dashed curve (corresponding to the case with a relativistic current but no boosts). All this is completely in line with previous results for the electromagnetic form factors [8].

## 4 Discussion

Considering the results for the axial form factor as presented in the previous Section, together also with the electromagnetic-form-factor results from a completely analogous study published in Ref. [8], one may wonder why predictions are obtained such that in all aspects they are readily in accordance with the experimental data and thus produce a consistent miscroscopic picture of the structure of the nucleons. This is especially remarkable in view of various previous attempts to explain the nucleon form factors (at low momentum transfers) from constituent quark models.

We argue for two main ingredients that are essential for achieving such results: firstly the strict observation of relativistic effects, secondly the usage of realistic quark model wave functions.

Certainly, confined few-quark systems have to be treated in a relativistic framework. For three-quark systems, such as the nucleons, it has been quite difficult so far to fulfill this demand, especially with regard to generating the wave functions and calculating covariant observables. The reasons are manifold and cannot all be discussed here. We have found that relativistic Hamiltonian dynamics provides a promising approach to account for relativistic effects in confined few-constituent-quark systems. Specifically the point-form version is well adapted for an exact calculation of the necessary boosts of the wave functions and, consequently, for a strictly covariant calculation of matrix elements for any observables. Poincaré-invariant quantum mechanics, in any of its formulations (such as point, instant, or front forms [9]), is rigorously defined on a Hilbert space of a finite number of particles. In this respect it is most appropriate for a relativistic treatment of constituent quark models, which, as effective models of QCD for low-energy hadrons, are also built with a finite number of degrees of freedom. In such a situation, relativistic Hamiltonian dynamics for few-particle problems lacks only the property of cluster separability [18–20]. However, for few-quark systems, it is legitimate to argue that this feature is not really important whenever the constituents cannot be separated asymptotically (as it is specifically the case for the nucleons as stable three-quark bound states).

The main characteristics of the GBE CQM have already been mentioned in the Introduction and expressed in much detail in several places in the literature [5,21]. With respect to the form factor results we emphasize only those properties that are essential. Obviously the baryon wave functions must have the correct spatial extensions and the required symmetries. The GBE CQM comes with a specific spin-flavor symmetry that arises from its theoretical foundation [5] and is constrained by a fit to baryon spectroscopy. The parametrization in Ref. [6] achieves a unified description of all light and strange baryon spectra in good agreement with phenomenological data. It turns out that also the eigenfunctions of the corresponding Hamiltonian are quite realistic. For the description of both the electromagnetic and axial structure of the nucleons rather subtle properties of the wave functions are essential. For instance, to simultaneously reproduce both the proton and neutron electromagnetic form factors in Ref. [8] as well as the axial form factor in the present work both the overall structure and the rather small mixed-symmetry configurations in the nucleon wave functions are crucial. Otherwise one would not obtain a consistent description without introducing further parameters beyond the quark model.

It is satisfying that in addition to these experimental constraints the GBE CQM also meets well-established properties of QCD in the low-energy regime. It respects the important consequences of SB$\chi$S (adressed in the Introduction) and is also consistent with the large $N_c$ behaviour of QCD, where the SU(6) symmetry of baryons becomes exact [22]. The particular confinement interaction used in the GBE CQM provides baryon wave functions with exactly this symmetry, while the GBE hyperfine interaction gives rise to the breaking of the SU(6) symmetry at lower order; for the nucleons, in particular, it produces the admixture of small mixed-symmetry components at order $1/N_c$, which is among the most important ingredients for the form factor results.

The present results have been obtained with a one-body current operator (in the point-form approach) only. In principle, one would also expect contributions from two-body operators. If they were large, they would spoil the good results. However, contributions from exchange-currents are intimately related to relativistic effects. In fact the PFSA used here is not the usual nonrelativistic impulse approximation, because the impulse given to the nucleon is not the same as the impulse given to the struck quark. Further, the one-body current is not only covariant, in the electromagnetic case it is also conserved. It remains to be seen by quantitative calculations how much possible remaining two-body currents will contribute. Since relativistic effects are fully accounted for in the point-form approach, there is good hope that beyond the PFSA any contributions from genuine two-body currents will remain small.

## 5 Summary

We have presented first results for the nucleon axial form factor predicted by the GBE constituent quark model of Ref. [6]. They were obtained in a covariant theory using the point-form approach of relativistic Hamiltonian dynamics. The full PFSA results, including all relativistic effects, are found to fall remarkably close to the experimental data at low and moderate momentum transfers without introducing any further parameters. This behaviour is in striking correspondence with the case of the electromagnetic observables considered so far for the GBE quark model; in Ref. [8] practically the same characteristics were found with regard to electromagnetic neutron as well as proton elastic form factors. It should be emphasized that the axial form factor also represents a stringent test for the quality of the nucleon wave functions. Again, as in the case of the electromagnetic observables, several realistic characteristics of the quark model wave functions, such as mixed-symmetry components or a proper size of the nucleons, are required to attain a reasonable result. The GBE quark model obviously provides such features for the baryon wave functions, in addition to producing the correct eigenenergies in the excitation spectra. All this indicates that the GBE interaction may be a reasonable phenomenological representation of the low-energy strong interaction.

In any case it has become obvious that relativistic effects are crucially important. Using Poincaré-invariant quantum mechanics appears to be a promising way of including them in a definite manner. In particular, the point-form approach makes it possible to reliably calculate relativistic effects for three-quark systems without the necessity of introducing any approximations.

## References


[1] Y. Nambu and G. Jona-Lasinio, Phys. Rev. **122** (1961) 345; ibid. **124** (1961) 246.

[2] J. Kokkedee, *The Quark Model* (Benjamin, New York, 1969).

[3] S. Aoki *et al.*, Phys. Rev. Lett. **82** (1999) 4392;
J. Skullerud and A. Williams, Phys. Rev. D **63** (2000) 054508.

[4] A. Manohar and H. Georgi, Nucl. Phys. B **234** (1984) 189.

[5] L.Ya. Glozman and D.O. Riska, Phys. Rep. **268** (1996) 263.

[6] L.Ya. Glozman, W. Plessas, K. Varga, and R.F. Wagenbrunn, Phys. Rev. D **58** (1998) 094030.

[7] R.F. Wagenbrunn, L.Ya. Glozman, W. Plessas, and K. Varga, Nucl. Phys. A **663**&**664** (2000) 703c; ibid. **666**&**667** (2000) 29c.

[8] R.F. Wagenbrunn, S. Boffi, W. Klink, W. Plessas, and M. Radici, Phys. Lett. B **511** (2001) 33.

[9] P.M.A. Dirac, Rev. Mod. Phys. **21** (1949) 392.

[10] B.D. Keister and W.N. Polyzou, Adv. Nucl. Phys. **20** (1991) 225.

[11] W.H. Klink, Phys. Rev. C **58** (1998) 3587.

[12] B. Bakamjian and L.H. Thomas, Phys. Rev. **92** (1953) 1300.

[13] W.H. Klink, Phys. Rev. C **58** (1998) 3617.

[14] K. Dannbom, L.Ya. Glozman, C. Helminen, and D.O. Riska, Nucl. Phys. A**616** (1997) 555.

[15] Particle Data Group, C. Caso *et al.*, Eur. Phys. J. C **3** (1998) 1.

[16] A. Liesenfeld *et al.*, Phys. Lett. B **468** (1999) 20.

[17] T. Kitagaki *et al.*, Phys. Rev. D **28** (1983) 436.

[18] S.N. Sokolov, Dokl. Akad. Nauk SSSR **233** (1977) 575 [Sov. Phys. Dokl. **22** (1977) 198].

[19] F. Coester and W.N. Polyzou, Phys. Rev. D **26** (1982) 1348.

[20] S. Weinberg, *The Quantum Theory of Fields*, Vol. 1 (Cambridge Univ. Press, Cambridge, 1995).

[21] L.Ya. Glozman, Nucl. Phys. A **663**&**664** (2000) 103.

[22] R. Dashen and A. Manohar, Phys. Lett. B **315** (1993) 425; E. Jenkins, Ann. Rev. Nucl. Part. Sci. **48** (1998) 81.